

Best Foot Forward or Best for Last in a Sequential Auction?*

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Abstract

Should an informed seller of multiple goods sell the best goods first to make a favorable impression on buyers, or instead wait until buyers have learned more from earlier sales? To help answer this question we consider the sequential auction of two goods by a seller with private information about their values. We find that the seller's sequencing strategy endogenously generates correlation in the values of the goods across periods, thereby giving the seller an incentive to impress buyers by leading with the better good. This impression effect implies that selling the better good first is the unique equilibrium in many situations, and that selling the better good last is never a unique equilibrium. However, if the seller could commit to a sequencing strategy, revenues would often be higher from waiting to sell the better good last. Either sequencing strategy reveals the seller's ranking of the goods and thereby, due to the linkage principle, generates higher revenues than randomly selling the goods or selling them simultaneously.

JEL Classification: D44, D82

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1 Introduction

Should the best or worst goods be sold first? This question arises whenever a seller has some private information about the quality of her goods and is concerned about the impact that early sales will have on buyer expectations. For instance, in privatization auctions should a government sell its most promising firms first to create a favorable impression on investors? Or should it warm up investors first with less valuable firms? Similarly, should a restructuring firm that is selling off multiple units start with the most profitable or least profitable ones? The traditional counsel to “put one’s best foot forward” might seem appropriate, but so does the recommendation to “save the best for last”.

To investigate this conflicting advice we consider an auction where a seller has private information about the values of two stochastically identical and *independently* distributed goods. We investigate the problem from two perspectives. First, from an equilibrium perspective, if buyers believe that the seller is leading with the better or worse good will the seller actually benefit from doing so, or will the seller prefer to fool buyers by reversing the order? Second, from a revenue-maximizing perspective, how do the equilibrium strategies rank in terms of revenues? And if the seller could commit to any sequencing strategy, what strategy would the seller choose?

We first consider the simpler case where the goods are auctioned simultaneously so that no information is released between auctions. In a simultaneous auction the equivalent of a sequencing strategy is for the seller to reveal which of the two goods is better.¹ We show that ranking the goods can be an equilibrium in that the good which buyers believe to be ranked higher according to the seller’s strategy is in fact the good that the seller has an incentive to rank higher. Regarding revenues, the ranking is an informative public signal about each good’s value, so the seller’s expected revenues are higher than if the seller randomly sold the better or worse good in either auction. This follows from the “linkage principle” (Milgrom and Weber, 1982a) which states that publicly revealing information equalizes the knowledge of buyers, thereby leading to more competitive bidding and higher expected revenues.

Using the simultaneous auction as a baseline, we then consider the sequential auction where one of the two goods is sold first. The difference is that buyers of the second period good now observe the first period price or other information about the first period good. For instance, in a privatization auction buyers of a firm sold later see the price of a firm sold first and, if the interval between auctions is sufficient, may also observe the post-privatization performance of the first firm. Because the values of the two goods are independently distributed, information from the first period may seem irrelevant for the second period good. However, we show that the seller’s sequencing strategy endogenously generates correlation across the two auction periods by truncating the distribution of the second period good. For instance, under a best foot forward strategy if the first period good receives a high price then second period buyers have some hope that the second period good is also of high quality. But if the first period

¹The ranking of goods in a simultaneous auction is examined in a pure common value context in Chakraborty, Gupta, and Harbaugh (2002). Here we consider the more general affiliated values case.

good receives a low price then second period buyers will conclude that if the first period good was of low quality there is even less hope for the second period good.

Because of this endogenous correlation between the first period signal and the value of the second period good, the seller has an incentive to strategically sequence the goods so as to make the most favorable impression on second period buyers. If buyers expect a best foot forward strategy, unexpectedly selling the worse good first will lead second period buyers to infer the second good is also low quality. So the “impression effect” from observing the first period signal penalizes deviation from the best foot forward strategy. But if buyers expect a best for last strategy, unexpectedly selling the better good first will lead second period buyers to think the second good is also high quality. So the impression effect encourages deviation from the best for last strategy. We find that the best foot forward strategy is always an equilibrium in the sequential auction whenever the best for last strategy is, and it is the unique pure strategy equilibrium when the impression effect is sufficiently strong.² Furthermore, the best foot forward strategy is always an equilibrium in the sequential auction whenever ranking the goods is an equilibrium in the simultaneous auction.

From a revenue perspective, a sequential auction has two advantages over a simultaneous auction. First, by expanding the region in which ranking the goods is an equilibrium, it facilitates credible revelation of the seller’s ordinal information. Second, the first period information is itself a public signal that, on average, increases the second period price in accordance with the linkage principle. The first period price will sometimes be high and sometimes be low with a corresponding upward or downward impact on the second period price, but on average the impact is positive. This effect has been previously noted in cases where the values of the two goods were identical (Milgrom and Weber, 1982c) or at least correlated (Hausch, 1986), but we show that endogenous correlation induces the same effect even when the two goods are *ex ante* independent.

While either sequencing strategy increases expected revenues by revealing the seller’s ordinal information and the first period price or other information, there may be tension between the sequencing strategy that is an equilibrium and the sequencing strategy that maximizes revenues. In particular, since best for last is never the unique pure strategy equilibrium, a conflict can arise between best for last as the revenue-maximizing strategy and best foot forward as the equilibrium strategy. In a parameterized example we show that expected revenues are slightly higher from the best for last strategy even though best foot forward is the unique pure strategy equilibrium. If the seller could commit to a strategy she would therefore choose best for last. But without commitment, best foot forward is the only credible sequencing strategy.

Regarding privatization auctions, governments usually sequence the sale of companies over many

²The impression effect has applications beyond auctions. For instance, when presenting two papers should the best or worst paper be presented first? If it is sufficiently important to the presenter that the audience for the best paper is largest, then either strategy is an equilibrium. But if the presenter just wants to maximize total attendance for the two papers, endogenous correlation and the impression effect imply that best foot forward is the unique pure strategy equilibrium.

years rather than following a “big bang” strategy of privatizing firms simultaneously (Roland, 2000). A common explanation for this delay is fear of selling firms at below market value. From the perspective of auction theory, this concern is justifiable because the lack of reliable public information about the firms can give buyers substantial information rents. This paper indicates that sequential privatization can increase revenues by credibly and publicly revealing information to all buyers, thereby reducing the value of private information and reducing buyer information rents.³ This same logic also applies to divestiture auctions.⁴ The restructuring literature has shown that the decision to sell assets implies information about both the value of the assets and the value of the remaining firm (Nanda, 1991). Our paper shows that, when multiple assets are sold, the sequencing decision also reveals information about the relative values of the assets. This information will decrease the expected price of one asset and increase the expected price of the other, but on average will increase revenues.

Sequencing strategies offer insight into the “declining price anomaly” or “afternoon effect” in which the prices for seemingly equivalent goods fall during the course of a sequential auction (Ashenfelter, 1989). Such declines have been observed in a wide range of environments, including privatization sales spread over many years (Lopez-de-Silanes, 1999). The phenomenon appears inconsistent both with the law of one price which states that arbitrage should ensure uniform prices, and with the result from auction theory that public information about bids in earlier auctions of identical goods should lead to rising prices due to the linkage principle (Milgrom and Weber, 1982c). Possible explanations for the anomaly include risk aversion (McAfee and Vincent, 1993), complementarities between goods (Branco, 1997; Menezes and Montiero, 1999), a declining number of bidders for later goods (Engelbrecht-Wiggans, 1994), auction participation costs (von der Fehr, 1994), moral hazard by agents bidding on behalf of clients (Ginsbergh, 1998), and special auction rules in which the winner can purchase additional units at the same price (Black and de Meza, 1992) or can choose her preferred item (Gale and Hausch, 1994). Our approach provides a simple explanation of this effect: prices fall because the quality of the second good is on average lower than the first good based on the seller’s private information. Under the best foot forward strategy we find that the negative effect on the second period price due to lower quality will outweigh the positive impact on that price due to the linkage principle, and thereby ensure a decline in prices.

Previous analyses of strategic sequencing consider the case where the goods are known to be different based on public information. Benoit and Krishna (2000) show in a complete information environment that leading with the better good maximizes revenue when buyers have budget constraints and act strategically across auctions. Bernhardt and Scoones (1994) consider a private value auction in which

³Consistent with the impression effect that favors a best foot forward strategy, empirical evidence from Gupta, Ham, and Svejnar (2000) shows that in the mass privatization programs undertaken in the Czech Republic more profitable firms were auctioned first. If the better firms are indeed sold first, then evaluation studies need to control for this selection bias to correctly estimate the gains from privatization.

⁴In a sample of divestitures in the 1989-1998 period, Boone and Mulherin (2002) find that, contrary to much of the discussion in the restructuring literature, auctions are a common selling mechanism.

the variance of buyer valuations differs between two goods and find that the good with highest variance should be sold first. Beggs and Graddy (1997) find that selling the best good first maximizes revenues when each buyer's private valuation for one good is a multiple of their valuation of the other and buyers demand only one good. Baba (1998) finds similar results under the weaker condition that buyer valuations are supermodular in the buyer's signal and the seller's signal. McMillan (1994) notes that the issue of whether to sell rights for large or small regions first was considered in designing spectrum auctions in the U.S., with one factor being that the linkage principle favored a small to large sequence. Our analysis differs from the previous literature in that the seller makes a sequencing decision based on her own *private* information. The sequencing strategy can therefore play a role in credibly (although, imperfectly) revealing this information to buyers.⁵

The paper proceeds as follows. Section 2 presents a simple example that illustrates the impression effect. Section 3 introduces the general auction model. Section 4 considers a simultaneous auction so as to analyze the simplest environment that supports rank-revealing strategies. Section 5 considers a sequential auction to see how the impression effect of observing information from the first period good affects the credibility of and revenues from sequencing strategies. It also analyzes the declining price anomaly. Section 6 presents an expanded example that illustrates the key findings. Section 7 concludes the paper and the Appendix contains most of the proofs.

2 An Introductory Example

The endogenous correlation generated by the seller's sequencing strategy, and the resulting impression effect, can be seen in a simple common value auction in which buyers do not have any private information so there are no information rents. Assume two ex ante identical goods, a and b , are sold in two periods by a seller who observes the actual values, V_a and V_b , of the goods. The goods are independently distributed with support in $\{0, 1\}$, and $\Pr[V_i = 0] = \Pr[V_i = 1] = \frac{1}{2}$ for $i = a, b$. There are two different groups of two or more identical buyers in each period. Buyers in the first period bid the expected value of the good conditional on the seller's strategy and buyers in the second period bid the expected value of the good conditional on both the seller's strategy and the observed value of the first good. Let V_1 represent the value of the good sold first and V_2 the value of the good sold second.

Figure 1 shows the distribution of values conditional on the seller's sequencing strategy.⁶ Consider the best foot forward strategy of leading with whichever good is better when $V_a \neq V_b$. If either $V_a = 1$ or $V_b = 1$ the first period good is expected to be high value so, based on these expectations and regardless

⁵In many auctions the goods are likely to differ based on both public information and the seller's private information, thereby representing a mix of previous models and our model. While additional issues are raised by such a mix, the role of the sequencing strategy in credibly revealing ordinal information, and the potential for conflict between equilibrium and revenue-maximizing strategies, remain as long as the seller has any private information.

⁶The correlation coefficient between V_1 and V_2 is $\rho = \frac{1}{3}$ in both cases. With binary values non-negative correlation is equivalent to affiliation, which is the basis for the theoretical results in the next section.

	$V_2 = 1$	$V_2 = 0$
$V_1 = 1$	$\frac{1}{4}$	$\frac{1}{2}$
$V_1 = 0$	0	$\frac{1}{4}$

Best Foot Forward

	$V_2 = 1$	$V_2 = 0$
$V_1 = 1$	$\frac{1}{4}$	0
$V_1 = 0$	$\frac{1}{2}$	$\frac{1}{4}$

Best For Last

Figure 1: Endogenous correlation from sequencing strategies

of what the seller actually does, the first period price is $\Pr[V_1 = 1] = \frac{3}{4}$. Now consider the impact on the second period price of either following the best foot forward strategy or deviating from it. If the seller follows the strategy then second period buyers observe that a high value good was sold in the first period, so the second period price is $\Pr[V_2 = 1|V_1 = 1] = \frac{1}{3}$. If instead the seller deviates and leads with the worse good then, expecting a best foot forward strategy, second period buyers will infer that if the first good was low value the second good must also be low value, so the second period price is $\Pr[V_2 = 1|V_1 = 0] = 0$. Since deviation is not profitable best foot forward is an equilibrium.⁷

It may seem that the problem is symmetric and best for last is also an equilibrium. Checking, buyers believe the first period good is high value only if both goods are high value so the first period price is $\Pr[V_1 = 1] = \frac{1}{4}$. If the seller follows the best for last strategy then second period buyers observe that a low value good was sold in the first period, so the second period price is $\Pr[V_2 = 1|V_1 = 0] = \frac{2}{3}$. However if the seller deviates and leads with the better good then the second period buyers believe the second good must also be of high value so the price is $\Pr[V_2 = 1|V_1 = 1] = 1$. Since deviation is profitable, best for last is not an equilibrium.

The seller wants to make a favorable impression even though goods a and b are independent because the seller's sequencing strategy endogenously generates correlation across the two periods by truncating the distribution of the second period good. As shown formally in the following sections, this intuition carries over to a more general information environment in which the seller need not be perfectly informed, the buyers have private information that may be stronger than that of the seller, the good has differing values to different buyers, and second period buyers observe only a noisy signal of the value of the first period good such as its price.

⁷We have ignored the cases where both goods are high or low value and the seller just randomizes. Adding these cases the expected payoff from sticking to best foot forward is $3/4 + (1/4)(1/3) + (1/2)(1/3) + (1/4)0 = 1$ and the expected payoff from deviating is $3/4 + (1/4)(1/3) + (1/2)0 + (1/4)0 = 5/6$.

3 The Model

We construct a standard auction model based on Milgrom and Weber (1982a) which includes both common value and private value features. As in their model, the seller and the buyers all have some private information. Distinct from their model, the seller can choose which of two goods to sell first based on her private information. This has two main implications when the seller follows a pure strategy of leading with the best or worst good. First, the fact that a good is sold in a particular period reveals the relative magnitude of the seller's signal for the good. Second, the endogenous correlation induced by the sequencing strategy implies information from the first period can be used to predict the seller's second period signal. For instance, observing the first period price reveals to second period buyers how highly buyers valued the first good, thereby giving an indication of the seller's signal for the first good and, due to the endogenous correlation, of the seller's signal for the second good. Disentangling the impact of these two sources of information will help us understand both the equilibrium and revenue-maximizing strategies. We set up our formal model below.

Goods, Signals & Values There is one seller who sells two goods indexed by $k \in \{a, b\}$. For each good k the seller observes a private signal $S_k \in \{H, L\} \subset \mathbb{R}$, where $H > L$. The seller can therefore tell if good k is likely to be above average ($S_k = H$) or below average ($S_k = L$) but nothing more.⁸ This information is soft in the sense that the seller cannot credibly reveal it, even though she may like to, except through the sequencing strategy. Let $\Pr[S_k = H] = \lambda \in (0, 1)$.

We suppose that for each good k there is a group of $n \geq 2$ buyers.⁹ The n buyers also observe private signals of the quality of the goods, $X_{ik} \in \mathbf{X} \subset \mathbb{R}$ for $i \in \{1, \dots, n\}$. We denote by $X_k = (X_{1k}, \dots, X_{nk})$ the vector of buyer signals for good k . For $j = 1, \dots, n$, let Z_{jk} be the j -th highest signal among the n signals of the buyers of good k with Z_k being the vector, and let Y_{ik}^j be the j -th highest signal of the bidders other than i with Y_{ik} being the vector.

We suppose that the random variables (X_k, S_k) associated with good k are independently and identically distributed across $k \in \{a, b\}$. To simplify notation, throughout the paper we use $f(\cdot)$ to represent densities and $f(\cdot|\cdot)$ to represent conditional densities. In particular, let $f(x, s)$ denote the joint density of (X_k, S_k) with support on $\mathbf{X}^n \times \{H, L\}$. We assume that the likelihood ratio $\frac{f(X_k|H)}{f(X_k|L)}$ of the conditional densities of X_k given S_k is bounded away from zero and infinity.¹⁰ Furthermore, following

⁸By limiting the seller's signal space we can restrict attention to the simplest and most intuitive sequencing strategies. When S has more than two elements (for example, S is a continuum) the seller could have more complicated strategies such as selling the better good first only if the gap between signals is sufficiently large.

⁹For simplicity we assume that there are different buyers for each good. This would be appropriate if, for instance, a government privatizes firms in different industries, or a restructuring firm sells off unrelated units. The assumption precludes such strategies as underbidding for the first good so as to lower other buyers' expectations for the second good (Hausch, 1986). It also prevents the buyer of the first good from acquiring an information advantage when bidding for the second good (Luton and McAfee, 1986).

¹⁰The bounded likelihood ratio assumption implies that buyers are never certain of the seller's information given their

Milgrom and Weber (1982a) we assume that the distribution of the buyers' private signals does not depend upon the identity of the buyers, or $f(x, s)$ is symmetric in its first n arguments, and that $f(x, s)$ displays affiliation. Affiliation implies that if one player (including the seller) observes a high private signal of the value of a good, other players are also more likely to observe high private signals of the value of that good. In particular it implies that the likelihood ratio above is non-decreasing in each argument.¹¹

We assume that for each good k there exists a function $V : \mathbf{X}^n \times \{H, L\} \rightarrow \mathbb{R}$, non-decreasing in its first n arguments and strictly increasing in its last argument, such that the value of good k to buyer i is given by $V_{ik} = V(X_{ik}, \{X_{i'k}\}_{i' \neq i}, S_k)$ for each buyer i and for each X_k and S_k . Since buyers are symmetric, the valuations of all buyers for good k depend on the seller's signal in the same way, and the valuation of each buyer depends on the signals of the other buyers $\{X_{i'k}\}_{i' \neq i}$ in the same way and does not depend on the identity of the other buyers.¹² Let $V_k = (V_{1k}, \dots, V_{nk})$ be the vector of buyer valuations for good k . Under our assumptions of affiliation and the monotonicity of the function $V(\cdot)$, the random variables (V_k, X_k, S_k) are affiliated as shown in Milgrom and Weber (1982a). Note that the joint density $f(v, x, s)$ of (V_k, X_k, S_k) is also distributed i.i.d. across k .

Seller's Strategies and the Timing Structure For a seller with signals $S_k = H$ and $S_{k'} = L$ where $k \neq k'$, the possible sequencing strategies are to sell the good with the high signal first (best foot forward or BFF strategy), to sell the good with the low signal first (best for last or BFL strategy) or to randomize (mixed strategy). When the signals for the two goods are identical the seller is indifferent about the sequencing strategy so that leading with either good is strategically equivalent and we allow the seller to randomize.¹³

We assume that the buyers in the second auction observe the first period price. We also allow for the possibility that the second period buyers observe other more informative public signals about the good sold in the first period. For example, the second period buyers in a privatization auction might observe the post-privatization performance of the firm sold in the first period.

Our primary interest is the sequential auction with the following timing structure: (1) The seller observes her signals and decides which good to sell first. (2) The buyers of the good the seller sells first note that their good is being sold first, observe their private signals, and bid for the good. (3) The buyers of the good the seller sells second note that their good is being sold second, observe their private signals, *observe the first period public signals relating to the first period good*, and bid for the good. For

own signal and so precludes the need to consider off-equilibrium-path beliefs.

¹¹Milgrom and Weber (1982a) provide a more extensive discussion of affiliation and its implications.

¹²As in Milgrom and Weber (1982a) this formulation allows for the possibility of pure common values, where $V_{ik} = V_k$ for all i . It also allows for a private value component for each buyer, so that buyers may not agree on value even if all private signals are made public.

¹³Strategies that condition on the names of the goods, such as selling good a first if the signals are the same and good b first otherwise, are not considered.

comparison we will also consider a simultaneous auction which is identical except that buyers of the “second” good do not receive any additional information regarding the first good.

Equilibrium We assume that the identical buyers for each good play a symmetric Bayesian Nash Equilibrium of the auction given their correct beliefs about the seller’s strategies and their information, and that the seller’s strategy is sequentially rational given the buyers’ beliefs.

The Auction, Prices and Bids We suppose that in each period the seller employs an English auction to sell the good.^{14,15} Notice from the timing structure above that, even for a fixed auction mechanism, the auctions in the two periods are different because the buyers may believe that the seller is treating the two periods differently via her sequencing strategy and because the second period buyers observe additional signals from the first period, e.g., the price. However, because of the symmetry in the model (including the seller’s strategies), the auction for each good k is identical. To exploit this symmetry, we will focus on the auction for each good k throughout our analysis.

In our model, in addition to their private signals X_k , there are a number of possible public signals that buyers of good k might receive. The first of these is the *period* in which good k is sold, denoted by $T_k \in \{\tau_H, \tau_L\}$, $\tau_H > \tau_L$. To illustrate, suppose that buyers believe that the seller is following a BFF strategy. Then, if good k is sold in the first period buyers in that period believe that $S_k = \max\{S_a, S_b\}$; whereas, if good k is sold in the second period then buyers of good k believe that $S_k = \min\{S_a, S_b\}$.¹⁶ The variable T_k indicates the public signal that buyers of good k receive from observing the period in which good k is sold:

$$\begin{aligned} T_k &= \tau_H \Rightarrow S_k = \max\{S_a, S_b\} \\ T_k &= \tau_L \Rightarrow S_k = \min\{S_a, S_b\}. \end{aligned}$$

The variable T_k is an ordinal signal that partially reveals the seller’s private information S_k and that captures the pure rank effect of the seller’s strategy. When the buyers believe that the seller is following the BFF (respectively, the BFL) strategy, the buyers of good k receive the signal τ_H when good k is sold in the first (respectively, the second) period, and receive the signal τ_L when good k is sold in the second (respectively, the first) period. We will denote by $P_k(X_k, T_k)$ the price of good k as a function of private and public signals when the only public signal that the buyers of good k receive is T_k .

¹⁴More precisely, we consider an ascending bid auction where the price rises continuously and each bidder has to decide when to drop out from the auction after observing the number of active bidders and when other bidders have dropped out. Drop-outs are final. See Milgrom and Weber (1982a).

¹⁵We do not consider reserve prices and entry fees. More generally, we do not consider mechanism design issues but instead take the selling mechanism as given.

¹⁶Symmetric remarks apply when the buyers believe that the seller will follow a BFL strategy.

In addition to the signal T_k , buyers of good k are also allowed to observe additional signals $\Psi_{k'}$ affiliated with the seller's signal $S_{k'}$ for good k' that is sold in the first period. For example, this could be the first period price $P_{k'}$ that the seller obtains for good k' . Notice that even though all signals associated with the two goods are independently drawn, the price $P_{k'}$ is correlated with the seller's signal for good k' and so, *via the seller's sequencing strategy*, it is correlated with the seller's signal for good k . As a result, the price $P_{k'}$ is relevant information for buyers of good k , as long as the buyers believe that the seller is conditioning her sequencing strategy on her private information. Furthermore, due to independence, the seller's sequencing strategy is the *only* channel through which the auctions for the two goods are related. We will denote by $P_k(X_k, T_k, \Psi_{k'})$ the price of good k as a function of private and public signals when buyers of good k receive the signal $\Psi_{k'}$ in addition to the signal T_k .

For any bidder i , and realizations $X_{ik} = x$, $Y_{ik}^j = y^j$ for $j = 1, \dots, n-1$, and $T_k = \tau$ define the function $v_k(\cdot)$ as

$$v_k(x, y^1, \dots, y^{n-1}, \tau) = E[V_{ik} | X_{ik} = x, Y_{ik}^1 = y^1, \dots, Y_{ik}^{n-1} = y^{n-1}, T_k = \tau]. \quad (1)$$

Further, for any realization $\Psi_{k'} = \psi$ define the function $v_k^*(\cdot)$ as

$$v_k^*(x, y^1, \dots, y^{n-1}, \tau, \psi) = E[V_{ik} | X_{ik} = x, Y_{ik}^1 = y^1, \dots, Y_{ik}^{n-1} = y^{n-1}, T_k = \tau, \Psi_{k'} = \psi]. \quad (2)$$

Note that due to our symmetry assumptions, neither $v_k(\cdot)$ nor $v_k^*(\cdot)$ depend on the identity of the bidder. From Milgrom and Weber (1982a), the bidder with the highest signal will win the auction and pay a price equal to the bid of the second highest bidder. The second highest bidder's bid will equal the expected value of the good given that he is tied for the highest bid after observing $n-2$ bidders with the lowest signals drop out (thereby inferring their signals Z_{3k}, \dots, Z_{nk}), and given the public signals T_k and $\Psi_{k'}$. As a result, the price $P_k(X_k, T_k)$ can be written as

$$P_k(X_k, T_k) = v_k(Z_{2k}, Z_{2k}, \dots, Z_{nk}, T_k) \quad (3)$$

while the price $P_k(X_k, T_k, \Psi_{k'})$ can be written as

$$P_k(X_k, T_k, \Psi_{k'}) = v_k^*(Z_{2k}, Z_{2k}, \dots, Z_{nk}, T_k, \Psi_{k'}). \quad (4)$$

4 Simultaneous Auction

Sequencing affects buyer information via the period in which a good is sold and the release of first period price or other first period information. To understand the impact on equilibrium strategies, we start by considering the case where buyers do not observe the first period price or any other information about the first auction. Without such information, the only public signal that the buyers of good k receive is the period signal T_k . Therefore this case is just a simultaneous auction where the "period" could be, for instance, the room that the good is auctioned in rather than the time that it is auctioned. Since the

two pure strategies BFF and BFL are identical subject to renaming the periods, we will refer to either strategy as a “rank-revealing strategy”. Such a strategy is credible if the seller has an incentive to sell the better good when buyers expect it to be sold rather than to deviate and trick the buyers.

For a seller with one high signal and one low signal,¹⁷ the symmetry between goods k and k' implies that the expected revenues from following a rank-revealing strategy are higher than from deviating if and only if

$$\begin{aligned} E[P_k(X_k, \tau_H)|S_k = H] + E[P_k(X_k, \tau_L)|S_k = L] &\geq \\ E[P_k(X_k, \tau_L)|S_k = H] + E[P_k(X_k, \tau_H)|S_k = L]. \end{aligned} \quad (5)$$

Our first proposition provides sufficient conditions on the primitives of the model under which a rank-revealing strategy is an equilibrium.

Proposition 1 *If*

$$V(x_1, \dots, x_n, H) - V(x_1, \dots, x_n, L) \text{ is non-decreasing in } x_1, \dots, x_n \quad (\text{ID})$$

then there exists $\bar{\lambda} \in (0, 1)$ such that if $\lambda \leq \bar{\lambda}$ a rank-revealing strategy is an equilibrium of the simultaneous auction.

Proof. See the Appendix. ■

The increasing differences condition (ID) implies that the seller’s signal and the buyers’ signals are complements in determining buyer valuations. Note that supermodularity of V in its arguments is equivalent to (ID). Intuitively, when the condition holds, buyers bid more aggressively when they are optimistic about the seller’s signal, so the seller should then sell her better good when buyers are expecting her to.

In a common value auction with a perfectly informed seller the difference (ID) reduces to just $H - L$ so the condition always holds. In affiliated values auctions, additive or multiplicative separability of buyer values in the seller’s information and the buyers’ own information is sufficient for (ID) to hold. Such separability can occur when the seller has information on a common component and the individual buyers have information on a private component. For instance, additive separability could arise in divestiture auctions of two units if buyers for each unit have a private signal about how well they could manage the unit, the divesting firm knows the general productivity of each unit, and profitability of a unit is an additive function of both factors. Multiplicative separability could occur in a privatization auction of two firms in separate industries if the buyers for each firm have a private signal about the firm’s profitability and the government knows the likely tax rates in the two industries.

Irrespective of whether a rank-revealing strategy equilibrium exists, it is easy to see that there is always a mixed strategy equilibrium where the buyers do not believe that the seller’s strategy reveals any information so the seller in fact sells either good first with probability $\frac{1}{2}$ regardless of S_a and S_b .

¹⁷When the seller receives identical signals, there is no strategic decision to be made as the seller is indifferent between sequencing choices, and we assume that, in equilibrium, the seller sells either good first with probability $\frac{1}{2}$.

Compared to the mixed strategy equilibrium, the rank-revealing strategy equilibrium yields the seller higher revenues. This follows from the linkage principle. For each good k , the “period” in which the good is sold is informative for the buyers as it tells them whether the seller’s signal S_k for the good is the maximum or the minimum of two independent draws. We state this as our next result.

Proposition 2 *A rank-revealing strategy generates higher expected revenues than the mixed strategy in the simultaneous auction.*

Proof. Follows immediately from Theorem 12, Milgrom and Weber (1982a). ■

Revealing seller information benefits the seller on average, but clearly the incentive to exaggerate makes direct statements about the value of the goods non-credible. Proposition 1 shows that in some cases the seller can credibly reveal ordinal information about the goods,¹⁸ and Proposition 2 shows that even this limited information increases expected revenues.

5 Sequential Auction

Now consider the impact of observing the first period price or other information from the first period auction. If buyers believe the seller follows either pure strategy, a high first period price raises the probability that the seller received two high signals, and therefore raises the estimated value of the second period good for each buyer. The first period price is more likely to be high when the good with the highest signal is sold in the first period since first period buyers are more likely to observe high private signals and bid correspondingly when the seller’s signal is high. We call the impact of observing first period information the “impression effect”.

Lemma 1 *For both the best foot forward and best for last strategies, when good k is sold in the second period, observation of the first period signal $\Psi_{k'}$ by second period buyers raises (lowers) the expected second period price if the seller sells a good with a high (low) signal $S_{k'}$ in the first period: for all $\tau \in \{\tau_H, \tau_L\}$ and each value $s \in \{H, L\}$ of S_k ,*

$$E[P_k(X_k, \tau, \Psi_{k'}) | S_{k'} = H, S_k = s] \geq E[P_k(X_k, \tau) | S_k = s] \geq E[P_k(X_k, \tau, \Psi_{k'}) | S_{k'} = L, S_k = s]. \quad (6)$$

Proof. In the Appendix. ■

The proof depends on the affiliation between the first period public signal $\Psi_{k'}$ and the first period seller signal $S_{k'}$, the independence between goods k and k' , the endogenous correlation between the first and second period seller signals, and the affiliation between the second period buyer signals and the second period seller signal. The lemma therefore holds for the special case where second period buyers observe only the first period price, $\Psi_{k'} = P_{k'}$.

¹⁸Chakraborty, Gupta, and Harbaugh (2002) show that in common value simultaneous auctions some ordinal information can always be revealed if the number of goods is large enough.

Corollary 1 *If buyers of good k only observe the first period price $P_{k'}$ which is affiliated with $X_{k'}$ and $S_{k'}$, then for all $\tau \in \{\tau_H, \tau_L\}$, and each value $s \in \{H, L\}$ of S_k ,*

$$E[P_k(X_k, \tau, P_{k'})|S_{k'} = H, S_k = s] \geq E[P_k(X_k, \tau)|S_k = s] \geq E[P_k(X_k, \tau, P_{k'})|S_{k'} = L, S_k = s]. \quad (7)$$

Proof. Follows analogously from the proof of the lemma. ■

If, as in the simultaneous auction, the first period price is not observed the best foot forward and best for last strategies are equivalent so either both are equilibria or neither are equilibria. The impression effect adds a boost in favor of the best foot forward strategy and against the best for last strategy, implying that the equilibrium condition for the former is always less strict than that for the latter. Thus whenever best for last is an equilibrium best foot forward must also be an equilibrium, but not the converse. One equilibrium which always exists is where the seller plays a mixed strategy of randomly sequencing the sale of the goods. If buyers expect such randomization the seller is indifferent between sequencing strategies because the first period price conveys no information.

Proposition 3 *(i) The equilibrium condition for best foot forward (best for last) is less (more) strict than the equilibrium condition for a rank-revealing strategy in the simultaneous auction. (ii) Best foot forward is an equilibrium whenever best for last is but not the converse. (iii) The mixed strategy is always an equilibrium.*

Proof. Using the symmetry between good k and good k' , the equilibrium condition for the BFF can be written as,

$$\begin{aligned} & E[P_k(X_k, \tau_H)|S_k = H] + E[P_k(X_k, \tau_L, \Psi_{k'})|S_{k'} = H, S_k = L] \\ \geq & E[P_k(X_k, \tau_H)|S_k = L] + E[P_k(X_k, \tau_L, \Psi_{k'})|S_{k'} = L, S_k = H] \end{aligned} \quad (8)$$

and that for the BFL strategy can be written as

$$\begin{aligned} & E[P_k(X_k, \tau_L)|S_k = L] + E[P_k(X_k, \tau_H, \Psi_{k'})|S_{k'} = L, S_k = H] \\ \geq & E[P_k(X_k, \tau_L)|S_k = H] + E[P_k(X_k, \tau_H, \Psi_{k'})|S_{k'} = H, S_k = L]. \end{aligned} \quad (9)$$

The proof of (i) follows from the lemma by observing that the left-hand side of (8) is greater than the left-hand side of (5) while the right-hand side of (8) is less than the right-hand side of (5), and similarly that the left-hand side of (9) is less than the left-hand side of (5) while the right-hand side of (9) is greater than the right-hand side of (5). The proof of (ii) then follows directly. The proof of (iii) follows from the fact that if the buyers believe that the seller does not condition the sequencing decision on her information then, for each realization of her signals, all sequencing strategies for the seller yield the same expected revenues. ■

The first part of this proposition implies that best foot forward is an equilibrium if (5) holds and that best foot forward is the unique pure strategy equilibrium if (5) holds with equality. More generally,

the existence and uniqueness of the best foot forward equilibrium will depend on the distribution of buyer signals, seller signals, and buyer valuations in a non-trivial way.¹⁹ Clearly, the more powerful is the impression effect, the more likely it is that the best foot forward strategy is the unique pure strategy equilibrium. The extreme case is where the seller is fully informed of the goods' values and the seller's first period signal is fully revealed. This situation was examined in the introductory example for a parameterized case where buyers did not have informative signals. It was found that best foot forward was the unique pure strategy equilibrium. In fact, this result also holds when buyers have informative signals.

Proposition 4 *In a common value auction where the seller is perfectly informed, $V_k = S_k$, and the seller's signal is revealed between periods, $\Psi_{k'} = S_{k'}$, best foot forward is an equilibrium and is the unique pure strategy equilibrium.*

Proof. Under the BFF strategy if the first period seller signal is L then buyers will infer the second period seller signal is also L . Therefore, for $\Psi_{k'} = L$, the second period price is $E[P_k(X_k, \tau_L, L)|S_{k'} = L, S_k = H] = L$. So the equilibrium condition for BFF simplifies to

$$E[P_k(X_k, \tau_H)|S_k = H] + E[P_k(X_k, \tau_L, H)|S_{k'} = H, S_k = L] \geq E[P_k(X_k, \tau_H)|S_k = L] + L \quad (10)$$

which holds since affiliation implies $E[P_k(X_k, \tau_H)|S_k = H] \geq E[P_k(X_k, \tau_H)|S_k = L]$ and since $V_k = S_k \in \{L, H\}$ implies $E[P_k(X_k, \tau_L, H)|S_{k'} = H, S_k = L] \geq L$. Conversely, under the BFL strategy $E[P_k(X_k, \tau_H, H)|S_{k'} = H, S_k = L] = H$. So the condition for BFL simplifies to

$$E[P_k(X_k, \tau_L)|S_k = L] + E[P_k(X_k, \tau_H, L)|S_{k'} = L, S_k = H] \geq E[P_k(X_k, \tau_L)|S_k = H] + H \quad (11)$$

which does not hold since affiliation implies $E[P_k(X_k, \tau_L)|S_k = L] \leq E[P_k(X_k, \tau_L)|S_k = H]$ and since $V_k = S_k \in \{L, H\}$ implies $E[P_k(X_k, \tau_H, L)|S_{k'} = L, S_k = H] < H$. ■

We now turn to the impression effect on seller revenues. Information from the first period provides additional information regarding the realization of the seller's signal in the second period. By the linkage principle the expected revenues from the best foot forward or best for last strategy are therefore higher when buyers observe first period information compared to the simultaneous auction. On the other hand, for the mixed strategy the expected revenues will be the same for the simultaneous and sequential auctions because, due to the independent values of the two goods, information about good k' contains no information for the buyers of good k since the seller does not condition the sequencing on her information. We collect these results into our next proposition.

Proposition 5 *(i) Both pure strategies in the sequential auction generate higher expected revenues than a rank-revealing strategy in the simultaneous auction. (ii) The mixed strategy generates the same expected revenues in the sequential and simultaneous auctions.*

¹⁹Note from Propositions 1 and 3(i) that the sufficient conditions on the primitives for the existence of a rank-revealing equilibrium for the simultaneous case are also sufficient for the existence of a BFF equilibrium.

Proof. Follows from the discussion above and Theorem 12 in Milgrom and Weber (1982a). ■

A natural question is whether the expected revenues from best foot forward and best for last can be unambiguously ranked. Under the best foot forward strategy the first period price signal is more likely to carry positive information about the second period good. However, under the best for last strategy the more valuable good is sold in the second period when this information has been released, thereby reducing buyer information rents for the more valuable good. Consequently, the revenues from either strategy may be higher and a conflict can arise between equilibrium and revenue-maximizing strategies if the best for last strategy yields higher revenues but is not an equilibrium.²⁰ In Section 6 we provide an example where best foot forward is an equilibrium and best for last is not, but best for last yields higher revenues.

To conclude this section we now turn to a discussion of expected prices across periods under the different pure strategies. Empirical evidence indicates that the prices of seemingly identical goods often fall during the course of a sequential auction. While a number of different approaches have been taken to explain this anomaly, they are focused primarily on buyer characteristics and strategies. We find that the “afternoon effect” can also arise endogenously out of the seller’s choice of an equilibrium sequencing strategy. Even though all signals related to the two goods are identically and independently distributed, prices are correlated over time. As the next result shows, when the seller employs the best foot forward strategy, prices fall simply because on average the second good is of lower value than the first good based on the seller’s private information. This negative effect outweighs the positive effect on second period prices from the linkage principle.²¹

Proposition 6 (i) *Under the best foot forward strategy the expected first period price is higher than the expected second period price:*

$$E[P_k(X_k, \tau_H)] \geq E[P_k(X_k, \tau_L, \Psi_{k'})]. \quad (12)$$

(ii) *Under the best for last strategy the expected first period price is lower than the expected second period price:*

$$E[P_k(X_k, \tau_L)] \leq E[P_k(X_k, \tau_H, \Psi_{k'})]. \quad (13)$$

(iii) *Under the mixed strategy the expected first period price is equal to the expected second period price.*

Proof. In the Appendix. ■

Note that in this model the afternoon effect does not imply violation of the law of one price. When good k is better based on the seller’s information it sells in the first period at a higher average price than good k' , but unconditional on the seller’s private information both goods sell at the same expected price.

²⁰When neither best foot forward nor best for last is an equilibrium then there is also a conflict since either strategy generates higher revenues than the mixed strategy.

²¹The proof relies on the independence of the valuations of the two goods a and b . If the valuations were sufficiently correlated the linkage principle’s effect could be dominant.

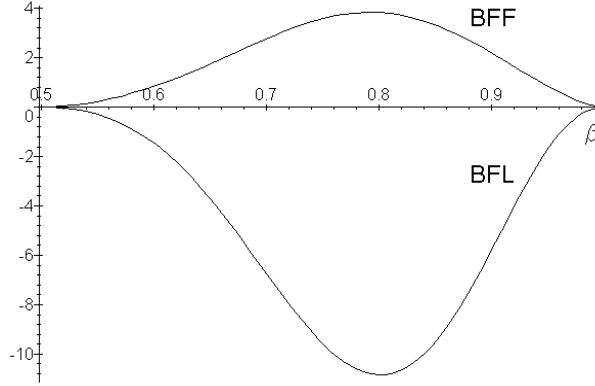


Figure 2: Percent revenue gain from following strategy vs. deviating, BFF and BFL

6 An Example

This section expands on the initial example of Section 2 by allowing buyers to have private information and by allowing the seller signal to be noisy. We consider the case where the impression effect is the weakest in that second period buyers only observe the first period price rather than a more informative signal of the first period good's value. In privatization and divestiture auctions, assets are typically sold over a period of many years so buyers interested in assets sold later are likely to see highly informative signals regarding the values of assets sold earlier.

As in the earlier example, $V_k \in \{0, 1\}$ where $\Pr[V_k = 1] = \frac{1}{2}$ for $k \in \{a, b\}$. Regarding the seller's signal $S_k \in \{L, H\}$, let

$$\Pr[S_k = H|V_k = 1] = \Pr[S_k = L|V_k = 0] = \alpha \in (\frac{1}{2}, 1] \quad (11)$$

for $k \in \{a, b\}$. Note that the signal is uninformative when $\alpha = \frac{1}{2}$. For each good there are $n = 2$ buyers who each receive a noisy binary signal of the quality of the good being sold in that period, $X_k \in \{L, H\}$, where

$$\Pr[X_k = H|V_k = 1] = \Pr[X_k = L|V_k = 0] = \beta \in (\frac{1}{2}, 1) \quad (12)$$

for $k \in \{a, b\}$. For simplicity we assume that the buyer and seller signals are independent conditional on the value of the good. We continue to use an English auction, which with two bidders is equivalent to a second-price auction.

First considering equilibrium strategies, in this example ranking the goods in the simultaneous auction is an equilibrium for $\lambda = \Pr[V_k = 1] \leq \frac{1}{2}$, and the seller is just indifferent for $\lambda = \frac{1}{2}$. Therefore for our case of $\lambda = \frac{1}{2}$ the impression effect implies that best foot forward is the unique pure strategy

equilibrium in the sequential auction.²² Figure 2 graphs expected revenue from following the candidate equilibrium strategy minus expected revenue from deviating as a percent of the latter when β varies from $\frac{1}{2}$ to 1 while α is fixed at 0.95. The top line shows that the seller benefits from sticking to the BFF strategy while the bottom line shows that the seller loses from sticking to the BFL strategy. Note that when β is very low the first period price provides no information to second period buyers and when β is very high second period buyers are so well informed that first period price information is redundant.²³ At either extreme the sequencing strategy is irrelevant since the impression effect of the first period price disappears.

Regarding revenue-maximizing strategies, Proposition 5 states that revenues from either the BFF or BFL strategies should be higher than those from the mixed strategy because the sequencing strategy and the first period price publicly reveal information. Since BFF is an equilibrium for all parameter values, the BFF strategy can credibly reveal seller information to buyers. In this example the revenue gain from either pure strategy relative to the mixed strategy is increasing in α and reaches a peak when β is about $\frac{3}{4}$. For low values of β neither buyer has much information while for high values of β both buyers have highly correlated information. The gains from reducing buyer information rents are therefore highest for intermediate values of β where these rents are largest. For $\alpha = .95$ and $\beta = \frac{3}{4}$, the revenue gain from BFF relative to the mixed strategy is about 7%. For all parameter values BFL offers slightly higher revenues than BFF, but since BFL is not an equilibrium so the seller will have difficulty committing to the strategy.

From Proposition 6 we know that the expected second period price is lower than the expected first period price when the seller follows the best foot forward strategy. Figure 3 shows this afternoon effect when α and β jointly vary from $\frac{1}{2}$ to 1. For $\alpha = \beta = \frac{1}{2}$ the seller's signal is completely uninformative so the buyers bid the unconditional expected value of $\frac{1}{2}$ in each period. As the informativeness of the seller's signal increases, the expected values of the goods diverge, with the expected value of the first good (labelled as $E[V_1]$) increasing linearly to $\frac{3}{4}$ and the expected value of the second good (labelled as $E[V_2]$) decreasing linearly to $\frac{1}{4}$. The expected prices for the two periods (labelled as $E[P_1]$ and $E[P_2]$) do not follow this pattern exactly due to the buyers' information rents. Although the information rents are smaller for the second good due to the linkage principle, as seen by the smaller gap between $E[V_2]$ and $E[P_2]$ than between $E[V_1]$ and $E[P_1]$, the expected prices decline between periods for all parameter values. If the two goods were identical, rather than just stochastically equivalent, the linkage principle

²²Recall that the incentive to follow BFF and deviate from BFL was stronger in the introductory example because the seller was perfectly informed ($\alpha = 1$) and the actual seller signal from the first period, rather than just the price, was revealed in the second period. In fact, from Proposition 4, under those assumptions BFF is the unique pure strategy equilibrium for any λ and any β .

²³ The pictured asymmetry between the gains from following BFF and the losses from following BFL arises because the English auction format makes a high price in the first period price a particularly strong signal of second period quality when buyers expect the seller to follow BFL. The incentive to deviate from BFL is therefore stronger than the incentive to follow BFF.

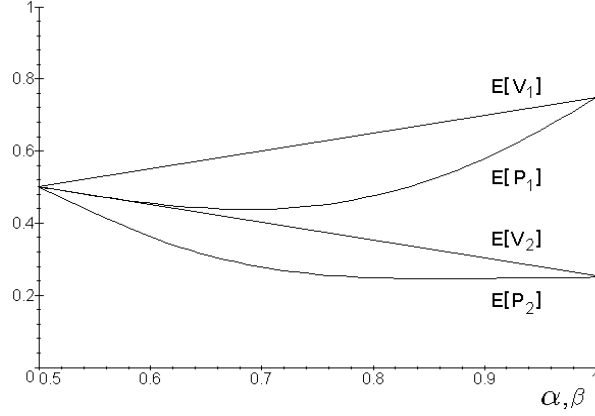


Figure 3: Declining price pattern under BFF strategy

would imply the opposite pattern of rising prices (Milgrom and Weber, 1982c).

7 Conclusion

This paper shows that sequencing is an important strategic decision in the auction of multiple goods even when the goods are ex ante independent. Leading with either the better or worse good endogenously generates correlation across periods so evidence of a high quality good in the first period, such as a high price, makes a positive impression on second period buyers. When the impression effect is strong enough, leading with the better good is the unique pure strategy equilibrium. Either strategy reveals the seller's private information about the relative profitability of the goods. Since this ordinal information is credible when the sequencing strategy is an equilibrium, revenues increase in accordance with the linkage principle.

The issue of how to credibly reveal ordinal information is more general than sequential auctions (Chakraborty, Gupta, and Harbaugh, 2002; Chakraborty and Harbaugh, 2003). While seller statements about the values of their goods are normally suspect, ordinal signals can be part of an equilibrium strategy since they simultaneously reveal both good and bad information. For instance, if a seller provides estimated valuations for a set of goods the ordinal information might be credible even if the cardinal information is not. Compared to simultaneous auctions, sequential auctions are better at revealing ordinal information even when the only signal between periods is the price. The impression effect from observation of the first period price expands the range in which a pure strategy equilibrium exists, and thereby expands the range in which the sequencing strategy can credibly reveal information.

8 Appendix

Proof of Proposition 1 Condition (5) is equivalent to showing that $E[P_k(X_k, \tau_H) - P_k(X_k, \tau_L) | S_k]$ is non-decreasing in S_k , which from (3) is equivalent to showing that $E[v_k(Z_{2k}, Z_{2k}, \dots, Z_{nk}, \tau_H) - v_k(Z_{2k}, Z_{2k}, \dots, Z_{nk}, \tau_L) | S_k]$ is non-decreasing in S_k . We look for sufficient conditions for this to hold.

From (1),

$$\begin{aligned} v_k(X_{ik}, Y_{ik}, T_k) &= E[V(X_{ik}, \{Y_{ik}^j\}, S_k) | X_{ik}, Y_{ik}, T_k] \\ &= \sum_{s \in \{H, L\}} V(X_{ik}, \{Y_{ik}^j\}, s) \Pr[S_k = s | X_{ik}, Y_{ik}, T_k]. \end{aligned}$$

Then

$$\begin{aligned} &v_k(X_{ik}, Y_{ik}, \tau_H) - v_k(X_{ik}, Y_{ik}, \tau_L) \\ &= \sum_{s \in \{H, L\}} V(X_{ik}, \{Y_{ik}^j\}, s) \{\Pr[S = s | X_{ik}, Y_{ik}, \tau_H] - \Pr[S = s | X_{ik}, Y_{ik}, \tau_L]\} \\ &= [V(X_{ik}, \{Y_{ik}^j\}, H) - V(X_{ik}, \{Y_{ik}^j\}, L)] \{\Pr[H | X_{ik}, Y_{ik}, \tau_H] - \Pr[H | X_{ik}, Y_{ik}, \tau_L]\}. \end{aligned} \quad (14)$$

From the properties of conditional probabilities,

$$\begin{aligned} \Pr[H | X_{ik}, Y_{ik}, \tau_H] &= \frac{f(H, X_{ik}, Y_{ik}, \tau_H)}{f(X_{ik}, Y_{ik}, \tau_H)} \\ &= \frac{\sum_{S_{k'} \in \{H, L\}} f(\tau_H | H, S_{k'}) f(X_{ik}, Y_{ik} | H) f(S_{k'}, H)}{\sum_{S_{k'} \in \{H, L\}} \sum_{S_k \in \{H, L\}} f(\tau_H | s, s') f(X_{ik}, Y_{ik} | S_k) f(S_{k'}, S_k)} \\ &= \frac{\lambda_1 f(X_{ik}, Y_{ik} | H)}{\lambda_1 f(X_{ik}, Y_{ik} | H) + (1 - \lambda_1) f(X_{ik}, Y_{ik} | L)} \end{aligned} \quad (15)$$

where

$$\lambda_1 \equiv \Pr[S_k = H | T_k = \tau_H] = 1 - (1 - \lambda)^2$$

and we have used the independence of signals associated with goods k and k' in the second line.

Similarly,

$$\Pr[H | X_{ik}, Y_{ik}, \tau_L] = \frac{\lambda_2 f(X_{ik}, Y_{ik} | H)}{\lambda_2 f(X_{ik}, Y_{ik} | H) + (1 - \lambda_2) f(X_{ik}, Y_{ik} | L)} \quad (16)$$

where

$$\lambda_2 \equiv \Pr[S_k = H | T_k = \tau_L] = \lambda^2.$$

Define the likelihood ratio of the densities of (X_{ik}, Y_{ik}) conditional on S_k as

$$l(X_{ik}, Y_{ik}) \equiv \frac{f(X_{ik}, Y_{ik} | H)}{f(X_{ik}, Y_{ik} | L)} \quad (17)$$

and define the function

$$h(l) \equiv \frac{2\lambda(1 - \lambda)l}{[\lambda_1 l + (1 - \lambda_1)][\lambda_2 l + (1 - \lambda_2)]}. \quad (18)$$

From (14)–(18),

$$E[\{V(Z_{2k}, Z_{2k}, \dots, Z_{nk}, H) - V(Z_{2k}, Z_{2k}, \dots, Z_{nk}, L)\}h(l(Z_{2k}, Z_{2k}, \dots, Z_{nk}))|S_k] \quad (19)$$

needs to be non-decreasing in S_k . Under condition (ID) the first expression in braces inside the expectation is non-decreasing in each of its last n arguments. Further, by affiliation, the likelihood ratio $l(Z_{2k}, Z_{2k}, \dots, Z_{nk})$ is also non-decreasing in each argument. By affiliation, it then suffices to show that $h(l)$ is non-decreasing in l . It can easily be verified that $h'(l) > 0$ if and only if

$$l < \sqrt{\frac{(1-\lambda)^2(1-\lambda^2)}{(1-(1-\lambda)^2)\lambda^2}}.$$

Let $\bar{\lambda}$ be such that

$$\sqrt{\frac{(1-\bar{\lambda})^2(1-\bar{\lambda}^2)}{(1-(1-\bar{\lambda})^2)\bar{\lambda}^2}} = \sup_{x, y^1, \dots, y^{n-1} \in \mathbf{X}^n} l(x, y^1, \dots, y^{n-1}).$$

Since the left-hand side of the expression above is continuous and monotonically decreasing in $\bar{\lambda}$, equal to 0 at $\bar{\lambda} = 1$, and approaching infinity as $\bar{\lambda}$ goes to 0, and since the right-hand side is bounded away from infinity by assumption, such a $\bar{\lambda}$ exists by the intermediate value theorem.

Proof of Lemma 1: Impression Effect Recall that, when the only signal that buyers of good k observe is T_k , the expected price of good k conditional on the seller's signal $S_k = s \in \{H, L\}$, given a realized value $\tau \in \{\tau_H, \tau_L\}$ of T_k is

$$E[P_k(X_k, \tau)|S_k = s] = E[v_k(Z_{2k}, Z_{2k}, \dots, Z_{nk}, \tau)|S_k = s].$$

Now, from (1) and (2)

$$\begin{aligned} v_k(X_{ik}, Y_{ik}^1, \dots, Y_{ik}^{n-1}, T_k) &= E[v_k|X_{ik}, Y_{ik}^1, \dots, Y_{ik}^{n-1}, T_k] \\ &= E[v_k^*(X_{ik}, Y_{ik}^1, \dots, Y_{ik}^{n-1}, T_k, \Psi_{k'})|X_{ik}, Y_{ik}^1, \dots, Y_{ik}^{n-1}, T_k]. \end{aligned}$$

By independence of the random variables related to good k from good k' the density of $\Psi_{k'}$ conditional on X_{ik}, Y_{ik}, T_k and $S_{k'}$ depends only on $S_{k'}$. Using this we obtain

$$v_k(X_{ik}, Y_{ik}^1, \dots, Y_{ik}^{n-1}, T_k) = E[E[v_k^*(X_{ik}, Y_{ik}^1, \dots, Y_{ik}^{n-1}, T_k, \Psi_{k'})|S_{k'}]|X_{ik}, Y_{ik}^1, \dots, Y_{ik}^{n-1}, T_k].$$

For realizations $X_{ik} = x, Y_{ik}^1 = y^1, \dots, Y_{ik}^{n-1} = y^{n-1}, T_k = \tau$, define the function

$$\hat{v}_k(x, y^1, \dots, y^{n-1}, \tau) \equiv E[E[v_k^*(x, y^1, \dots, y^{n-1}, \tau, \Psi_{k'})|S_{k'}]|X_{ik} = x, Y_{ik}^1 = y^1, \dots, Y_{ik}^{n-1} = y^{n-1}, T_k = \tau].$$

By affiliation of $S_{k'}$ with $\Psi_{k'}$ and the monotonicity of v_k^* in $\Psi_{k'}$ we have

$$\begin{aligned} &E[v_k^*(x, y^1, \dots, y^{n-1}, \tau, \Psi_{k'})|S_{k'} = H] \\ &\geq \hat{v}_k(x, y^1, \dots, y^{n-1}, \tau) \\ &\geq E[v_k^*(x, y^1, \dots, y^{n-1}, \tau, \Psi_{k'})|S_{k'} = L]. \end{aligned}$$

Therefore,

$$\begin{aligned}
& E[P_k(X_k, \tau) | S_k = s] \\
&= E[\widehat{v}_k(Z_{2k}, Z_{2k}, \dots, Z_{nk}, \tau) | S_k = s] \\
&\geq E[E[v_k^*(Z_{2k}, Z_{2k}, \dots, Z_{nk}, \tau, \Psi_{k'}) | S_{k'} = L] | S_k = s] \\
&= E[v_k^*(Z_{2k}, Z_{2k}, \dots, Z_{nk}, \tau, \Psi_{k'}) | S_{k'} = L, S_k = s] \\
&= E[P_k(X_k, \tau, \Psi_{k'}) | S_{k'} = L, S_k = s].
\end{aligned}$$

Similarly,

$$\begin{aligned}
& E[P_k(X_k, \tau) | S_k = s] \\
&= E[\widehat{v}_k(Z_{2k}, Z_{2k}, \dots, Z_{nk}, \tau) | S_k = s] \\
&\leq E[E[v_k^*(Z_{2k}, Z_{2k}, \dots, Z_{nk}, \tau, \Psi_{k'}) | S_{k'} = H] | S_k = s] \\
&= E[v_k^*(Z_{2k}, Z_{2k}, \dots, Z_{nk}, \tau, \Psi_{k'}) | S_{k'} = H, S_k = s] \\
&= E[P_k(X_k, \tau, \Psi_{k'}) | S_{k'} = H, S_k = s].
\end{aligned}$$

This concludes the proof. ■

Proof of Proposition 6: Afternoon Effect For realizations $X_{ik} = x, Y_{ik}^1 = y^1, \dots, Y_{ik}^{n-1} = y^{n-1}, T_k = \tau$ and $S_{k'} = s'$, define the functions

$$v_k^{**}(x, y^1, \dots, y^{n-1}, \tau, s') \equiv E[V_{ik} | X_{ik} = x, Y_{ik}^1 = y^1, \dots, Y_{ik}^{n-1} = y^{n-1}, T_k = \tau, S_{k'} = s'] \quad (20)$$

and

$$v_k^m(x, y^1, \dots, y^{n-1}) \equiv E[V_{ik} | X_{ik} = x, Y_{ik}^1 = y^1, \dots, Y_{ik}^{n-1} = y^{n-1}]. \quad (21)$$

We start with the proof of (i). Note first that the expected second period price under the BFF strategy is higher when second period buyers actually directly observe the seller's first period signal than when they only observe the first period price:

$$E[P_k(X_k, \tau_L, \Psi_{k'})] \leq E[v_k^{**}(Z_{2k}, Z_{2k}, \dots, Z_{nk}, \tau_L, S_{k'})] \quad (22)$$

as the signal $\{\tau_L, S_{k'}\}$ contains more information about S_k than the signal $\{\tau_L, \Psi_{k'}\}$ (see Theorem 13 in Milgrom and Weber (1982a)).

Furthermore, for fixed x, y^1, \dots, y^{n-1} and $\tau = \tau_L$,

$$\begin{aligned}
v_k^{**}(x, y^1, \dots, y^{n-1}, \tau_L, s') &= E[V_{ik} | X_{ik} = x, Y_{ik}^1 = y^1, \dots, Y_{ik}^{n-1} = y^{n-1}, T_k = \tau_L, S_{k'} = s'] \\
&\leq E[V_{ik} | X_{ik} = x, Y_{ik}^1 = y^1, \dots, Y_{ik}^{n-1} = y^{n-1}, S_{k'} = s'] \\
&= E[V_{ik} | X_{ik} = x, Y_{ik}^1 = y^1, \dots, Y_{ik}^{n-1} = y^{n-1}] \\
&= v_k^m(x, y^1, \dots, y^{n-1})
\end{aligned}$$

where the inequality follows from affiliation of T_k with the other random variables related to good k , the next equality follows from the fact that $S_{k'}$ is independent of S_k (and contains information about S_k only in conjunction with τ_k) and the last equality is definitional. Thus,

$$E[v_k^{**}(Z_{2k}, Z_{2k}, \dots, Z_{nk}, \tau_L, S_{k'})] \leq E[v_k^m(Z_{2k}, Z_{2k}, \dots, Z_{nk})]. \quad (23)$$

Finally, note that

$$E[P_k(X_k, \tau_H)] = E[v_k(Z_{2k}, Z_{2k}, \dots, Z_{nk}, \tau_H)] \geq E[v_k^m(Z_{2k}, Z_{2k}, \dots, Z_{nk})] \quad (24)$$

as

$$\begin{aligned} v_k(z_2, z_2, \dots, z_n, \tau_H) &= E[V_{ik} | X_{ik} = z_2, Y_{ik}^1 = z_2, \dots, Y_{ik}^n = z_n, T_k = \tau_H] \\ &\geq E[V_{ik} | X_{ik} = z_2, Y_{ik}^1 = z_2, \dots, Y_{ik}^n = z_n] \\ &= v_k^m(z_2, z_2, \dots, z_n) \end{aligned}$$

where the inequality follows from affiliation. From (22)—(24) we conclude that (i) holds.

The proof of (ii) is similar and that of (iii) follows immediately from symmetry and the inability of the mixed strategy to reveal information. ■

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